

What did we learn about GPDs from hard exclusive electroproduction of mesons?

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Abstract

It is reported on an analysis of electroproduction of light mesons at small Bjorken- x (x_{Bj}) within the handbag approach. The partonic subprocesses, meson electroproduction off quarks or gluons, are calculated within the modified perturbative approach (m.p.a.) in which quark transverse momenta are retained. The soft hadronic matrix elements, generalized parton distributions (GPDs), are constructed by means of double distributions. The constraints from parton distributions and sum rules are taken into account. Various moments of these GPDs are compared to recent results from lattice gauge theories.

Keywords: Handbag factorization, generalized parton distributions, electroproduction

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It has been shown [1] that, at large photon virtuality Q^2 , meson electroproduction factorizes in partonic subprocesses, electroproduction off gluons or quarks, $\gamma^* g(q) \rightarrow M g(q)$, and in GPDs, representing soft proton matrix elements which encode the soft, non-perturbative physics. The calculation of the subprocess amplitudes requires the meson's wave function, i.e. a second soft, non-perturbative function. It has also been shown that in this so-called handbag approach which offers a partonic description of meson electroproduction, the dominant contribution is generated by transitions from longitudinally polarized virtual photons (γ_L^*) to like-wise polarized vector mesons (V_L). Other transitions are suppressed by inverse powers of the large scale, Q^2 , although, as we had to learn, they are not small at experimentally accessible values of Q^2 . Thus, for instance, the ratio of the longitudinal over the transverse cross sections for the production of ρ^0 or ϕ mesons is about 2 for $Q^2 \simeq 4 \text{ GeV}^2$. Other clear signals for contributions from transverse photons come from asymmetries measured in π^+ electroproduction with a transversely polarized target [2] and from the transverse cross section for this process measured by the $F_\pi - 2$ collaboration [3].

In the following it is reported on an analysis [4, 5] of exclusive meson electroproduction within the handbag factorization scheme carried through in the kinematical regime of large energy in the photon-proton center of mass frame ($W \geq 4 \text{ GeV}$), low skewness ($\xi \simeq x_{\text{Bj}}/(2 - x_{\text{Bj}}) \leq 0.1$) and small momentum transfer ($-t \leq 0.6 \text{ GeV}^2$). In this kinematical region the dominant helicity amplitudes for the process $\gamma^* p \rightarrow V p$ where V denotes a vector meson, read

$$\begin{aligned} \mathcal{M}_{\mu+, \mu+}^V &= \frac{e_0}{2} \sum_a e_a C_V^a \left\{ \langle H \rangle_{V\mu}^g + \langle H \rangle_{V\mu}^a + \langle \tilde{H} \rangle_{V\mu}^g + \langle \tilde{H} \rangle_{V\mu}^a \right\}, \\ \mathcal{M}_{\mu-, \mu+}^V &= -\frac{e_0}{2} \frac{\sqrt{-t}}{2m} \sum_a e_a C_V^a \left\{ \langle E \rangle_{V\mu}^g + \langle E \rangle_{V\mu}^a \right\}. \end{aligned} \quad (1)$$

Explicit helicity labels refer to the proton while $\mu (= 0, \pm 1)$ denotes the helicity of the photon and the vector meson. The quark flavors are denoted by a and e_a is the corresponding charge. The non-zero flavor weight factors read for the vector mesons of interest

$$C_{\rho^0}^u = -C_{\rho^0}^d = C_\omega^u = C_\omega^d = 1/\sqrt{2}, \quad C_\phi^s = 1. \quad (2)$$

The terms $\langle F \rangle$ denote convolutions of subprocess amplitudes and GPDs ($F = H, E, \tilde{H}$) for the two relevant subprocesses, $\gamma^* g \rightarrow V g$ and $\gamma^* q \rightarrow V q$. Explicitly the convolutions read ($i = g, a$, $x_g = 0$, $x_a = -1$)

$$\langle F \rangle_{V\mu}^i = \sum_\lambda \int_{x_i}^1 dx \mathcal{H}_{\mu\lambda, \mu\lambda}^{Vi}(x, \xi, Q^2, t = 0) F^i(x, \xi, t). \quad (3)$$

The helicity of the parton is labeled by λ . Note that $\langle \tilde{H} \rangle_V^i = 0$ for longitudinal photons. The GPD \tilde{E} contributes to the transverse amplitudes only to order ξ and is consequently neglected as is the contribution from E to the proton helicity non-flip amplitudes because it is proportional to ξ^2 .

The subprocess amplitudes \mathcal{H} are calculated within the modified perturbative approach [6] in which quark transverse degrees of freedom as well as Sudakov suppressions are taken into account. This factorization scheme is based on work by [7, 8]. It is assumed in [4, 5] that the quarks and gluons are emitted and re-absorbed by the proton collinearly. The quark transverse momenta, k_\perp , are only taken into account in the subprocesses. This approximation is justified by the fact that the r.m.s. k_\perp of the partons inside the proton is much smaller (the GPDs describe the full proton) than that in the meson for which only the rather compact valence Fock state is considered. Since the Sudakov factor is known only in the impact parameter space [6], canonically conjugated to the transverse momentum space, the subprocess amplitudes are calculated in the b space

$$\mathcal{H}_{\mu\lambda,\mu\lambda}^{Vi} = \int d\tau d^2b \hat{\Psi}_{V\mu}(\tau, -\mathbf{b}) \hat{\mathcal{F}}_{\mu\lambda,\mu\lambda}^i(x, \xi, \tau, Q^2, \mathbf{b}) \alpha_S(\mu_R) \exp[-S(\tau, \mathbf{b}, Q^2)]. \quad (4)$$

Their t dependences are neglected for consistency since they provide corrections of order t/Q^2 which are generally neglected. On the other hand, the t dependence of the GPDs is taken into account since this t is scaled by a soft parameter. The hard scattering kernels \mathcal{F} , or their respective Fourier transform $\hat{\mathcal{F}}$, are calculated to leading-order of perturbative QCD including quark transverse momenta. The explicit expressions can be found in [4]. Also for the Sudakov factor S in (4) and the choice of the renormalization (μ_R) and factorization (μ_F) scales it is referred to these articles. In collinear approximation the amplitudes for transversely polarized photons and vector mesons suffer from infrared singularities [9, 10]. The quark transverse momenta, \mathbf{k}_\perp , provide an admittedly model-dependent regularization scheme of these singularities by replacements of the type

$$\frac{1}{dQ^2} \longrightarrow \frac{1}{dQ^2 + \mathbf{k}_\perp^2} \quad (5)$$

in the parton propagators. Here, d is a momentum fraction or a product of two. As can be readily shown the transverse amplitudes are suppressed by $\langle k_\perp^2 \rangle^{1/2}/Q$ as compared to the longitudinal ones in this regularization scheme.

The (longitudinal) amplitudes for electroproduction of pions are analogous to (1) with the replacement of H and E by \tilde{H} and \tilde{E} , respectively. Of course, the gluonic subprocess is not allowed in this case. For the case of π^+ production pion exchange is to be taken into account as well.

The GPDs are constructed with the help of double distributions [11]. The main advantage of this construction is the guaranteed polynomiality of the GPDs. The double distribution is written as a product of a zero-skewness GPD and a weight function that generates the skewness dependence of the full GPD ($n_g = n_{\text{sea}} = 2$, $n_{\text{val}} = 1$)

$$f_i(\beta, \eta, t) = F_i(\beta, \xi = 0, t) \frac{\Gamma(2n_i + 2)}{2^{2n_i+1} \Gamma^2(n_i + 1)} \frac{[(1 - |\beta|)^2 - \eta^2]^{n_i}}{(1 - |\beta|)^{2n_i+1}}. \quad (6)$$

The zero-skewness GPD is parameterized as

$$F_i(\beta, \xi = 0, t) = e^{b_i t} |\beta|^{-\alpha_i' t} h_i(\beta). \quad (7)$$

The function h_i represents the forward limit, $\xi = t = 0$, of the GPD. For H and \tilde{H} the forward limits are the phenomenologically known unpolarized and polarized PDFs, respectively. They have to be suitably continued to negative values of β . For the other GPDs the forward limits are parameterized as

$$h_i(\beta) = N_i \beta^{\alpha(0)} (1 - \beta)^\gamma. \quad (8)$$

As is well-known, at low β the PDFs behave power-like where the powers are given by the intercepts of appropriate Regge trajectories. It seems plausible to generate also the t -dependence of the GPDs

by Regge ideas and to assume that such a Regge-like behavior holds for the other GPDs as well. Assuming linear Regge trajectories $\alpha_i(t) = \alpha_i(0) + \alpha'_i t$ ($i = g, \text{ sea, valence}$) and exponential t -dependencies of the Regge residues, one arrives at the parameterization (7). For large $-t$ one likely needs a more complicated t dependence [12].

The full GPDs are obtained by an integral over f_i

$$F^i(x, \xi, t) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\eta \delta(\beta + \xi\eta - x) f_i(\beta, \eta, t). \quad (9)$$

There are other methods to generate the skewness dependence, namely the Shuvaev transform [13] and the dual parameterization [14]. Both these methods lead to very similar results for the GPDs at small skewness.

In Ref. [4] the Regge parameters are fixed in the following way: In agreement with the HERA data [15] on the integrated cross section σ_L and with the CTEQ6 PDFs [16] the gluon ('Pomeron') trajectory¹ is taken as $\alpha_g = 1.10 + 0.06 \ln(Q^2/4 \text{ GeV}^2) + 0.15 \text{ GeV}^{-1}t$. The increase of its intercept with Q^2 is a consequence of evolution. Since the sea quarks mix with the gluons under evolution, $\alpha_{\text{sea}}(t) = \alpha_g(t)$ is assumed. For the valence quark GPDs, H and E , on the other hand, a standard Regge trajectory is taken - $\alpha_{\text{val}} = 0.48 + 0.90 \text{ GeV}^{-1}t$. For the other GPDs there is no prominent Regge exchange, probably Regge cuts also play an important role. Therefore, effective trajectories are used to parameterize the low x behavior of these GPDs [5] whose parameters are fitted to experiment as it is done for the slope parameters b_i .

It has been checked in [4, 5] that the GPDs respect positivity bounds as well as the sum rules, i.e. their first moments are in agreement with the nucleon form factor data at small $-t$. The forward limit of E for the valence quarks is chosen in agreement with the form factor analysis performed in [12].

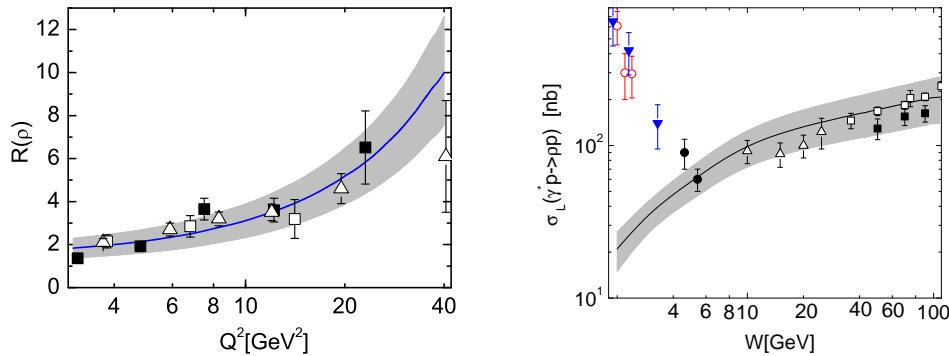


Figure 1: Left: Handbag predictions [4] of the ratio of longitudinal and transverse cross sections for ρ^0 production versus Q^2 at $W = 90 \text{ GeV}$ shown as a solid line. Data taken from H1 (solid squares) and ZEUS (open squares and triangles). Right: Predictions of the longitudinal cross section of ρ^0 production versus W at $Q^2 = 4 \text{ GeV}^2$. The open circles represent the recent CLAS data [17], the solid triangles the CORNELL data [18]. The shaded bands indicate the uncertainties of the theoretical analysis. For detailed references it is referred to [4].

Given that E is not much larger than H and \tilde{H} much smaller (see below) the cross sections for vector meson electroproduction are dominated by contributions from the GPD H at small skewness and small $-t$. An exception is ρ^+ production for which the relevant flavor combination $E_v^u - E_v^d$ is indeed substantially larger than $H_v^u - H_v^d$ (v denotes valence quarks). The GPD E is for instance probed by transverse target asymmetries which are related to interference terms $\text{Im}[\langle E \rangle^* \langle H \rangle]$, and \tilde{H} by double spin asymmetries like A_{LL} and by electroproduction of pions. In [4] a detailed analysis of cross sections and spin density matrix elements for ρ^0 and ϕ electroproduction

¹Note that the forward limit of the gluonic GPD H is defined as $xg(x)$ and analogously for the other GPDs.

has been performed in the kinematical range of $W \simeq 5 - 180 \text{ GeV}$ and $Q^2 \simeq 3 - 100 \text{ GeV}^2$. Generally very good results have been obtained. Two example of the results obtained in [4] are shown in Fig. 1. The ratio of the longitudinal and transverse cross sections increases $\propto Q^2$ due to the power suppression of the transverse amplitude. Despite this behavior the ratio is not large for Q^2 less than about 10 GeV^2 indicating a substantial contribution from the transverse amplitude. The longitudinal cross section for ρ^0 production shows a mild increase at large W since $\sigma_L \propto W^{4(\alpha_g(0)-1)}$. This signals the dominance of the gluonic subprocess (with a certain admixture from sea quarks). The valence quarks are only perceptible for W smaller than about 10 GeV . For HERMES kinematics ($W \simeq 5 \text{ GeV}$) the gluon (plus sea) contribution still amounts to about 50% of the cross section. Inspection of Fig. 1 reveals on the other hand that the handbag approach as proposed in [4] fails for low W : The sharp increase of σ_L between $W = 5$ and 2 GeV [17, 18] is not reproduced if the cross section is evaluated from the above described low-skewness GPDs simply extrapolated to larger ξ and E being still neglected. The reason of this discrepancy is still unclear. In contrast to this result an analogous extrapolation to low W for ϕ production leads to fair agreement with experiment [19]. This may be regarded as a hint at a small gluonic (and sea quark) E .

In Fig. 2 the transverse target asymmetry for ρ^0 production is shown. The main contribution to it comes from an interference of E for valence quarks and H for gluons (plus sea). The zero-skewness GPD E , see (7), is taken from [12] (with $\alpha_{\text{val}}, \gamma_e^u = 4.0$ and $\gamma_e^d = 5.6$). Given the errors of the HERMES data [20] a reasonable fit to experiment (solid line) is obtained if E^g and E^{sea} are ignored². The other theoretical curves in Fig. 2 represent results for various variants of E where E^g and E^{sea} are estimated from positivity bounds and a combination of Ji's sum rule and the momentum sum rule of deep inelastic lepton-nucleon scattering [12, 22]. At present only extreme variants seem to be excluded. It is to be emphasized that with E and H at disposal one can evaluate Ji's sum rule. It turns out [23] that the total angular momenta of u quarks and gluons are large while those of d and strange quarks are very small. The results for u and d quarks are in very good agreement with lattice gauge theory [24].

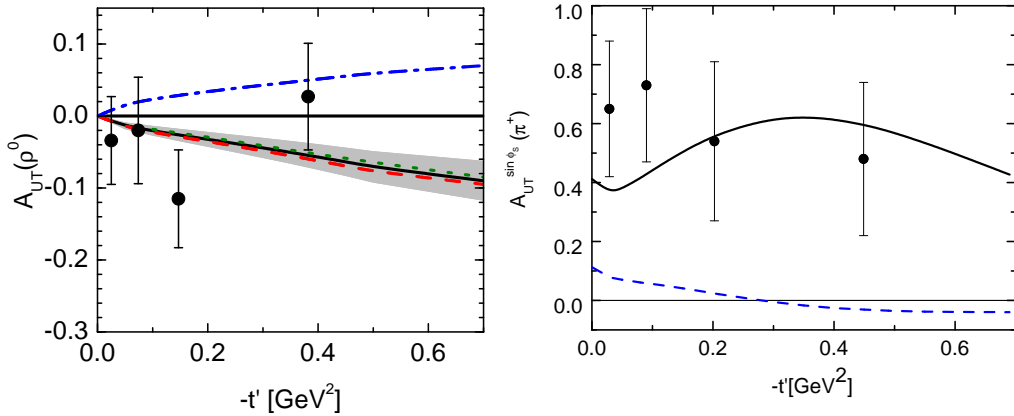


Figure 2: Left: The asymmetry A_{UT} for ρ^0 at $Q^2 = 3 \text{ GeV}^2$ and $W = 5 \text{ GeV}$. Right: The $\sin \phi_s$ moment for π^+ electroproduction at $Q^2 = 2.45 \text{ GeV}^2$ and $W = 3.99 \text{ GeV}$. The solid lines represent the predictions from the handbag approach [4, 5, 23]. Data taken from [20, 25].

The HERMES data on the cross section and the transverse target asymmetries for π^+ production have been analyzed in [5]. The relevant GPDs are \tilde{H} and \tilde{E} as well as pion exchange. However, this is not all. The $\sin \phi_s$ moment measured with a transversely polarized target [25] is very large and does not seem to vanish for forward scattering, see Fig. 2. Such a behavior can

² Note that there are also preliminary data on this observable from COMPASS [21]. Both the sets of data together favor negative values of A_{UT} .

GPD	probed by	constraints	status
H	ρ^0, ϕ cross sections	PDFs	known
\tilde{H}	-	polarized PDFs	probably small
E	$A_{UT}(\rho^0, \phi)$	sum rule for 2^{nd} moment	probably small
others	-	-	unknown
H	ρ^0, ϕ cross sections	PDFs, Dirac ff	known
\tilde{H}	π^+ data	pol. PDFs, axial ff	known
E	$A_{UT}(\rho^0, \phi)$	Pauli ff	known
$\tilde{E}^{n.p.}$	π^+ data	-	uncertain
H_T	π^+ data	transversity PDFs [29]	known
others	-	-	unknown

Table 1: Status of small-skewness GPDs as extracted from meson electroproduction data. The upper part is for gluons and sea quarks, the lower part for valence quarks. Except of H for gluons and sea quarks all GPDs are probed for scales of about 4 GeV^2 .

only be generated by the interference of the two helicity non-flip amplitudes

$$A_{UT}^{\sin \phi_s} \sim \text{Im}[\mathcal{M}_{0-,++}^* \mathcal{M}_{0+,0+}] . \quad (10)$$

As has been advocated in [5] the amplitude $\mathcal{M}_{0-,++}$, describing a $\gamma_T \rightarrow \pi$ transition, can be modeled within the handbag approach as a twist-3 effect combining the leading-twist helicity-flip GPDs [26] with the twist-3 pion wave function [27]. Taking into account only the most important helicity-flip GPD, namely H_T , one has

$$\mathcal{M}_{0-,++}^{\text{twist-3}} = e_0 \langle H_T^u - H_T^d \rangle . \quad (11)$$

The convolution is to be calculated with the subprocess amplitude $\mathcal{H}_{0-,++}$ which is parametrically suppressed by μ_π/Q as compared to $\mathcal{H}_{0+,0+}$. The parameter μ_π takes on a value of about 2 GeV at a scale of 2 GeV . Hence, the twist-3 effect is sizeable for Q of the order of a few GeV . With this twist-3 effect a good description of all HERMES data has been achieved in [5].

The GPDs extracted from meson electroproduction data in Refs. [4, 5, 23] are valid for $\xi \lesssim 0.1$ and are probed by experiment for $x \lesssim 0.6$. The present status of these GPDs is summarized in Tab. 1 and the valence quark GPDs are displayed in Fig. 3.

Since their parameterizations have no nodes except at the end-points and since they have similar t dependences, their well-known lowest moments at $t = 0$ fix the relative signs and strength of these GPDs.

Comparison with recent lattice QCD studies [24, 28] where the lowest moments of the GPDs $H, \tilde{H}, E, \tilde{E}$ and H_T for u and d quarks have been calculated, reveals that in general there is good agreement with the relative strength of moments and their relative t dependences. At small t even the absolute values of the moments agree quite well but the t dependence of the moments obtained from lattice QCD are usually flatter than the form factor data and the moments evaluated from the GPDs. An exception is the lowest moment of H_T for u quarks for which a value that is about 25% smaller than the lattice result has been found in [5]. In Fig. 4 the axial-vector and the pseudoscalar form factors are shown as examples. The form factors evaluated from the GPDs are compared to experimental data [30, 31] and to results from lattice QCD. For the pseudoscalar form factor only the pion-pole contribution

$$F_P^{\text{pole}}(t) = 4m^2 g_A [1 - t/\Lambda_N^2]^{-1} [m_\pi^2 - t]^{-1} \quad (12)$$

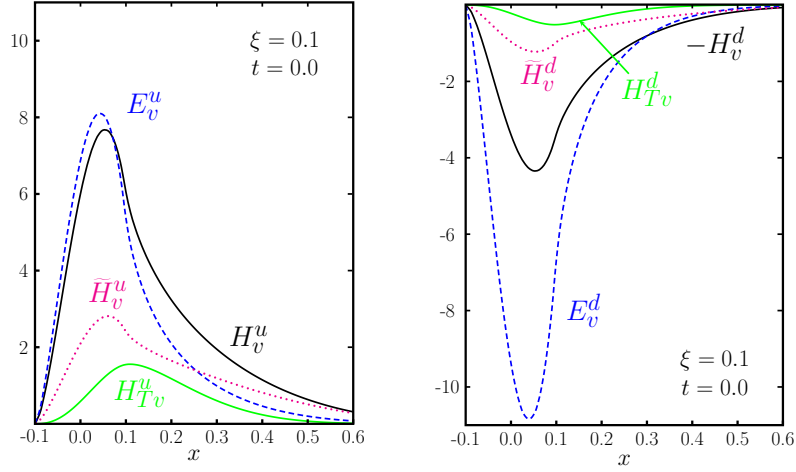


Figure 3: The valence-quark GPDs versus x at $\xi = 0.1$ and $t = 0$. The scale is 4 GeV^2 .

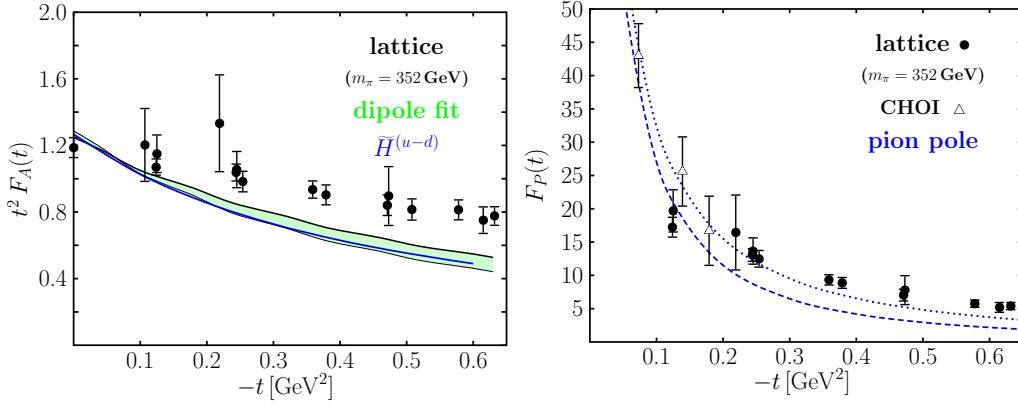


Figure 4: Left: The axial form factor of the nucleon scaled by t^2 . The green band represents the dipole fit to the data [30], the solid circles the lattice results [24] for $m_\pi = 352 \text{ MeV}$. The thick solid line is the form factor evaluated from \tilde{H} . Right: The pseudoscalar form factor of the nucleon. Experimental data from [31], lattice results from [24]. The dashed (dotted) line represents the pion-pole contribution with $\Lambda_N = 0.51(0.8) \text{ GeV}$.

is shown for two values of the parameter Λ_N . Since one may also expect a flat behavior for this form factor from lattice QCD there is some room left for non-pole contributions from \tilde{E} at large $-t$.

In summary the handbag approach proposed in [4, 5, 23] which consists of GPDs constructed from double distributions and power corrections generated from quark transverse momenta in the subprocess describes quite well the data on meson electroproduction measured by HERMES, COMPASS, FNAL and HERA over a wide range of kinematics. In this report the present status of this analysis is summarized and described what we have learned about the GPDs from it. In order to improve the GPDs more polarization data and data on π^0 electroproduction are required.

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References

- [1] A.V. Radyushkin, Phys. Lett. **B385**, 333 (1996);
J.C. Collins *et al.*, Phys. Rev. D **56**, 2982 (1997).
- [2] A. Airapetian *et al.* [HERMES Collaboration], Phys. Lett. **B682**, 345 (2010).
- [3] H. P. Blok *et al.* [Jefferson Lab Collaboration], Phys. Rev. **C78** (2008) 045202.
- [4] S. V. Goloskokov and P. Kroll, Eur. Phys. J. **C42**, 281 (2005); *ibid.* **C53**, 367 (2008).
- [5] S. V. Goloskokov and P. Kroll, Eur. Phys. J. **C65**, 137 (2010).
- [6] J. Botts and G. Sterman, Nucl. Phys. B **325**, 62 (1989).
- [7] R. K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. B **207**, 1 (1982).
- [8] J. C. Collins and D. E. Soper, Nucl. Phys. B **194**, 445 (1982).
- [9] L. Mankiewicz and G. Piller, Phys. Rev. D **61**, 074013 (2000).
- [10] I. V. Anikin and O. V. Teryaev, Phys. Lett. B **554**, 51 (2003).
- [11] D. Müller *et al.*, Fortschr. Phys. **42**, 101 (1994);
A.V. Radyushkin, Phys. Lett. **B449**, 81 (1999).
- [12] M. Diehl, T. Feldmann, R. Jakob and P. Kroll, Eur. Phys. J. **C39**, 1 (2005).
- [13] A. D. Martin *et al.*, Eur. Phys. J. C **63** (2009) 57.
- [14] M. V. Polyakov and K. M. Semenov-Tian-Shansky, Eur. Phys. J. A **40**, 181 (2009).
- [15] F. D. Aaron *et al.* [H1 Collaboration], JHEP **1005**, 032 (2010).
- [16] J. Pumplin *et al.*, JHEP **0207**, 012 (2002).
- [17] S. A. Morrow *et al.* [CLAS Collaboration], Eur. Phys. J. A **39**, 5 (2009).
- [18] D. G. Cassel *et al.*, Phys. Rev. D **24**, 2787 (1981).
- [19] J. P. Santoro *et al.* [CLAS Collaboration], Phys. Rev. C **78**, 025210 (2008).
- [20] A. Airapetian *et al.* [HERMES Collaboration], Phys. Lett. **B679** (2009) 100.
- [21] A. Sandacz, presented at the workshop on 'Electr. Inter. with Nucleons and Nuclei', Milos (2007).
- [22] M. Diehl and W. Kugler, Eur. Phys. J. **C39**, 933 (2007).
- [23] S. V. Goloskokov and P. Kroll, Eur. Phys. J. **C59** (2009) 809.
- [24] Ph. Hagler *et al.* [LHPC Collaborations], Phys. Rev. **D77**, 094502 (2008).
- [25] A. Airapetian *et al.* [HERMES Collaboration], Phys. Lett. **B682**, 345 (2010).
- [26] M. Diehl, Eur. Phys. J. **C19**, 485 (2001); P. Hoodbhoy *et al.*, Phys. Rev. D **58**, 054006 (1998).
- [27] V. M. Braun and I. E. Halperin, Z. Phys. **C48**, 239 (1990) [Sov. J. Nucl. Phys. **52**, 126 (1990)].
- [28] M. Gockeler *et al.* [QCDSF and UKQCD Collaboration], Phys. Lett. B **627**, 113 (2005).
- [29] M. Anselmino *et al.*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009).
- [30] T. Kitagaki *et al.*, Phys. Rev. **D28**, 436 (1983).
- [31] S. Choi *et al.*, Phys. Rev. Lett. **71**, 3927 (1993).